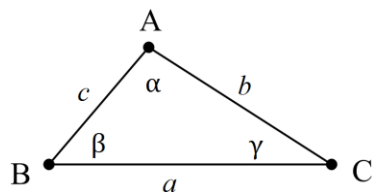


The Law of Sines

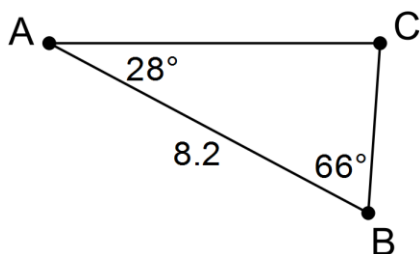
Solving a triangle means finding the measures of all the sides and angles. An **oblique triangle** is a triangle without a right angle. To solve an oblique triangle, we must know three pieces of information, at least one of which must be the length of a side. (Three angles define an infinite number of triangles).



The Law of Sines: In any triangle, $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$

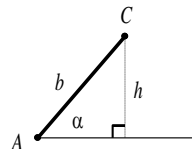
ASA or AAS: Find the third angle using the fact that the three angles of a triangle add to 180° . Then use the law of sines to find the other two sides of the triangle.

Example: $\alpha = 28^\circ$, $\beta = 66^\circ$, $c = 8.2$

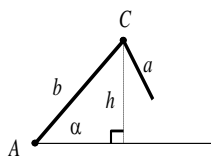


SSA (The Ambiguous Case): If you know two sides and a non-included angle (an angle that is not between the sides), there may be zero, one, or two possible triangles that fit the given measurements. To figure out how many triangles there are for an *acute* angle α , do the following:

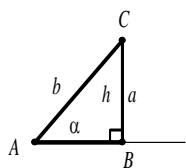
1. Draw the given angle (α) in standard position with a terminal side of length b . Don't draw side a yet.
2. Let h be an altitude from C to the initial side of α .



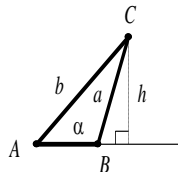
3. Since $\sin \alpha = h/b$, $h = b \sin \alpha$. Compare h to a as follows:
 - a. If $a < h$, then no triangle can be formed.
 - b. If $a = h$, then one triangle (a right triangle) can be formed.
 - c. If $h < a < b$, then two triangles can be formed.
 - d. If $a \geq b$, then one triangle can be formed.



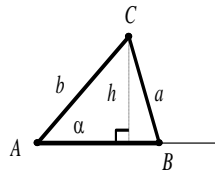
(a) $a < h$
no triangle



(b) $a = h$
one triangle

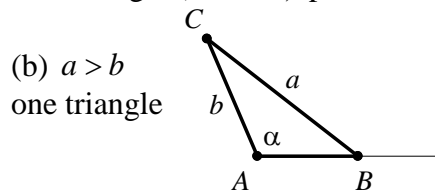
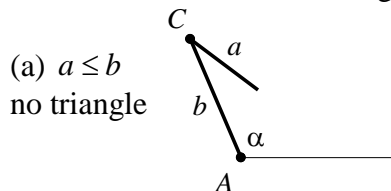


(c) $h < a < b$
two triangles



(d) $a \geq b$
one triangle

If α is *obtuse*, there is either no triangle (if $a \leq b$) or one triangle (if $a > b$) possible.



Examples:

a) $\beta = 38^\circ$, $b = 2.9$, $c = 5.9$

b) $\beta = 38^\circ$, $b = 6.4$, $c = 5.9$

c) $\beta = 38^\circ$, $b = 4.7$, $c = 5.9$

d) $\gamma = 125^\circ$, $b = 5.7$, $c = 8.6$